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Cold dark matter models with a cosmological constant

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ABSTRACT
We use linear and quasi-linear perturbation theory to analyse cold dark matter models of structure formation in spatially flat models with a cosmological constant. Both a tilted spectrum of density perturbations and a significant gravitational wave contribution to the microwave anisotropy are allowed as possibilities. We provide normalizations of the models to microwave anisotropies, as given by the four-year *COBE* observations, and show how all the normalization information for such models, including tilt, can be condensed into a single fitting function which is independent of the value of the Hubble parameter. We then discuss a wide variety of other types of observations. We find that a very wide parameter space is available for these models, provided Ω_0 is greater than about 0.3, and that large-scale structure observations show no preference for any particular value of Ω_0 in the range 0.3 to 1.

Key words: cosmology: theory – dark matter.

1 INTRODUCTION

Even before the *COBE* satellite's detection of microwave background anisotropies, a popular strategy for dealing with the perceived ills of the standard cold dark matter (CDM) model of structure formation was to reduce the matter density. This shifts the redshift of matter–radiation equality, distorting the shape of the spectrum of density perturbations to provide a good fit to the galaxy correlation function. In order to retain compatibility with typical models of inflation, a flat universe is desired which can be brought about by adding a cosmological constant Λ to the model (Peebles 1984; Turner, Steigman & Krauss 1984; Efstathiou, Sutherland & Maddox 1990; Kofman, Gnedin & Bahcall 1993; Stompor, Górski & Banday 1995; Klypin, Primack & Holtzman 1995). We will denote such models ΛCDM models.

The ΛCDM models are most commonly considered in the context of a scale-invariant (Harrison–Zel'dovich) primordial spectrum of density perturbations (Efstathiou et al. 1990; Stompor et al. 1995; Klypin et al. 1995), corresponding to spectral index $n = 1$. Another limitation usually placed on the models is that the gravitational waves should not contribute significantly to the large-scale microwave background anisotropy. However, within a given model of inflation both the spectral index and the gravitational wave contribution are determined, and while many models do give n very close to 1 and negligible gravitational waves, there are other models that do not. It is therefore realistic to allow both n and the gravitational wave contribution to vary when confronting a model with the data, and we will adopt that viewpoint in this paper. Observational constraints on n and the gravitational wave contribution will in the future place powerful constraints on models of inflation, and hence on the nature of the fundamental interactions at very high-energy scales. Kofman et al. (1993) have previously considered one example of the model with (mild) tilt and a gravitational wave contribution. Because the ΛCDM model has generated further interest recently (Krauss & Turner 1995; Ostriker & Steinhardt 1995; Bagla, Padmanabhan & Narlikar 1995), we believe that the full parameter space merits detailed study, and in this paper we compare a variety of predictions of the model, accessible via linear and quasi-linear perturbation theory, with observations of various types.

We thus permit arbitrary variation of three main parameters, namely the matter density Ω_0 expressed as a fraction of the critical density (the assumption always being that a cosmological constant restores spatial flatness), the present value of the Hubble parameter $H_0 = 100\,h\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$, and the spectral index n of the primordial spectrum. In addition, we will consider two possibilities concerning gravitational waves; first that they are negligible, and secondly that they have an amplitude appropriate to the power-law inflation model.

One of the most important observations is that of cosmic microwave background (CMB) anisotropies by the *COBE* satellite, and we provide careful normalizations of the models to the four-year *COBE* data. We quote a new fitting function which gives accurate normalizations for any combination of parameters. We also provide some comparison with intermediate-scale anisotropies, where the obser-

vational situation is less clear. We then go on to address a variety of constraints concerning large-scale structure, extending techniques we have already employed for critical-density models (Liddle et al. 1996b) and for open universe CDM models (Liddle et al. 1996a). The most closely related paper is Stompor et al. (1995), who provided extensive comparison with *COBE* for the case of a scale-invariant ($n = 1$) initial power spectrum, along with discussion of other large-scale structure constraints. They did not, however, discuss tilt or gravitational waves; nor did they attempt to delineate the allowed parameter region in the Ω_0–h plane.

2 THE POWER SPECTRUM

We specify the power spectrum of the density contrast, following the notation of Liddle & Lyth (1993) and Liddle et al. (1996a), as

$$\mathcal{P}_\delta(k) = \left(\frac{k}{aH}\right)^4 \delta_{\rm H}^2(k)\, T^2(k)\, \frac{g^2(\Omega)}{g^2(\Omega_0)}\,, \qquad (1)$$

where k is the wave number, a is the scale factor, $H = \dot{a}/a$ is the Hubble parameter and Ω is the density parameter. Subscript '0' indicates the present value. One can think of the remaining terms in three parts. First, $\delta_{\rm H}^2(k)$ gives the primordial spectrum of anisotropies, before they are affected by any evolution. The shape is determined by whichever mechanism is used to create the spectrum, which we will assume is cosmological inflation. If $\delta_{\rm H}^2(k)$ is a constant then one has the Harrison–Zel'dovich spectrum, while $\delta_{\rm H}^2(k) \propto k^{n-1}$ corresponds to a 'tilted' spectrum with spectral index n. Such a spectrum is the generic prediction of slow-roll inflation (Liddle & Lyth 1992). The amplitude of $\delta_{\rm H}^2(k)$ fixes the present-day normalization of the spectrum. The transfer function $T(k)$ contains the evolution of the spectrum from its primordial form to its present form, and depends on cosmological parameters and the nature of the matter in the universe. It approaches unity on large scales (which retain their primordial form), and in a CDM model is time-independent well after matter–radiation equality. Finally, there is a factor which specifies the time, or redshift, dependence of the amplitude of the spectrum at late epochs. If Ω_0 equals 1 this is carried in the $(aH)^4$ prefactor; for low-density cosmologies there is an additional suppression factor which we parametrize via a quantity $g(\Omega)$, defined below.

A quantity related to the power spectrum is the dispersion $\sigma(R)$ of the density field smoothed on a (comoving) scale R, defined by

$$\sigma^2(R) = \int_0^\infty W^2(kR)\, \mathcal{P}_\delta(k) \frac{{\rm d}k}{k}\,. \qquad (2)$$

To carry out the smoothing, we will always use a top-hat window function $W(kR)$ defined by

$$W(kR) = 3\left(\frac{\sin(kR)}{(kR)^3} - \frac{\cos(kR)}{(kR)^2}\right)\,. \qquad (3)$$

For CDM models as considered here, the transfer function is accurately given by Bardeen et al. (1986) as

$$T_{\rm CDM}(q) = \frac{\ln(1+2.34q)}{2.34q} \times \left[1 + 3.89q + (16.1q)^2 + (5.46q)^3 + (6.71q)^4\right]^{-1/4}\,, \qquad (4)$$

with $q = k/h\Gamma$, where the 'shape parameter' Γ is defined as (Sugiyama 1995)

$$\Gamma = \Omega_0 h \exp(-\Omega_{\rm B} - \Omega_{\rm B}/\Omega_0)\,. \qquad (5)$$

Here $\Omega_{\rm B}$ is the baryon density, which we take to equal $0.016h^{-2}$ as suggested by recent analyses of nucleosynthesis (Copi, Schramm & Turner 1995a,b). This fit to Γ is good for both flat and open universes (unless Ω_0 is very low).

We have accounted for the redshift dependence of the amplitude of the power spectrum by introducing a growth suppression factor $g(\Omega)$, following Carroll, Press & Turner (1992). This gives the total suppression of growth of the dispersion $\sigma(R)$ relative to that of a critical-density universe, and is accurately parametrized by

$$g(\Omega) = \frac{5}{2}\Omega\left[\frac{1}{70} + \frac{209\Omega}{140} - \frac{\Omega^2}{140} + \Omega^{4/7}\right]^{-1}\,. \qquad (6)$$

This formula can be applied at any value of Ω. For a matter-dominated flat universe, the redshift dependence of Ω is given by

$$\Omega(z) = \Omega_0 \frac{(1+z)^3}{1 - \Omega_0 + (1+z)^3\Omega_0}\,. \qquad (7)$$

Since the growth law in a critical-density universe is $\sigma(z) \propto (1+z)^{-1}$ (carried by the $(aH)^4$ term in equation (1)), the redshift dependence for arbitrary Ω_0 is therefore

$$\sigma(R,z) = \sigma(R,0)\,\frac{g(\Omega(z))}{g(\Omega_0)}\,\frac{1}{1+z}\,. \qquad (8)$$

3 MICROWAVE BACKGROUND ANISOTROPIES

3.1 *COBE*

With the detection of large-angle temperature fluctuations in the cosmic microwave background, the *COBE* satellite made possible for the first time an accurate normalization of models of structure formation. At the large scales probed the spectrum can be normalized in a regime where the theory is well understood, circumventing complications introduced by processing of the primordial spectrum and the relationship between observed structure and the underlying density field.

The first-year *COBE* data were of low signal-to-noise ratio, with the rms fluctuation having a 30 per cent error. Fits to the full data set, the angular correlation function or the rms fluctuation all gave consistent values for the quadrupole normalization: $Q_{\rm rms-PS} = 17 \pm 5\,\mu$K (Smoot et al. 1992; Seljak & Bertschinger 1993; Scaramella & Vittorio 1993; Wright et al. 1994).

The second year of data (Bennett et al. 1994) resulted in a dramatic improvement of the signal-to-noise ratio and a consequent increase in the degree of refinement of the analyses. The two-year data constrain the large-scale normalization to within 10 per cent (at 1σ), with 5–7 per cent of this being due to irremovable cosmic and sample variance. Ironically, along with the better signal-to-noise ratio came ambiguity in the number to use for normalization (amounting to a 30 per cent discrepancy!), due in large part to a low quadrupole in the two-year map (Banday et al. 1994; Bunn, Scott & White 1995). Normalization of models directly to the temperature maps became essential to obtain

all the information available from the *COBE* data (Górski et al. 1994; Bond 1995; Bunn 1995). In addition, highly accurate calculations of theoretical predictions and their relation to large-scale structure were included in the analyses (Bunn et al. 1995; Bunn & Sugiyama 1995; Hu, Bunn & Sugiyama 1995a; Górski et al. 1995; Tegmark & Bunn 1995; White & Bunn 1995; Stompor et al. 1995; Yamamoto & Bunn 1995; Cayon et al. 1996; White & Scott 1996), making the *COBE* normalization one of the most accurately known pieces of information about large-scale structure.

With the release of the four-year anisotropy maps (Bennett et al. 1996; Banday et al. 1996; Górski et al. 1996; Hinshaw et al. 1996), we have in hand all of the knowledge about large angular scale anisotropies that we can expect to obtain in the near future. The full data set prefers a slightly lower normalization ($\sim 1\sigma$) than the two-year data, due in equal parts to a statistical downward fluctuation and an improved galaxy cut (Górski et al. 1996). Models with near scale-invariant spectra, such as the ΛCDM models considered here, are now a better fit to the data and the actual quadrupole on the sky is no longer anomalously low. However, while the 'final' normalization is lower than that of the two-year data, the central value is still higher than that which would be obtained from considering only the rms fluctuation in the map. The *COBE* data still cannot be summarized by one number and fits to the full data set are necessary to obtain a precise normalization.

For low-density CDM models, the *COBE* data play a vital role in breaking the degeneracy between flat ΛCDM models and open CDM models. While arguments based on structure formation alone are insensitive to a cosmological constant (Martel & Wasserman 1990; Martel 1991; Lahav et al. 1991), the *COBE* normalizations of the two theories are very different. We will say more on this in the conclusions.

Inflation predicts not only a spectrum of density perturbations, but also one of gravitational waves (also known as tensor perturbations). Those with a wavelength comparable to the size of the observable Universe can induce extra microwave background anisotropies over and above those caused by the density perturbations. In absolute generality, the contribution of these to the *COBE* normalization is independent of the spectral index of density perturbations, permitting an arbitrary reduction in the normalization of the density perturbations. For $n < 1$, we choose to examine two cases. The first case has negligible gravitational waves, which is a prediction of many inflationary models even when they give significant tilt (the reason being that the gravitational waves are negligible unless the potential during inflation is within a few orders of magnitude of the Planck scale). In the second, the gravitational wave amplitude is the one given by power-law inflation (PLI); this happens to be a good approximation for the known models of inflation that give a significant gravitational wave contribution. We will also study $n > 1$, but in that case we will not consider gravitational waves since it is hard to arrange gravitational waves of significant amplitude when n is greater than 1.

For our assumed inflationary theories the density perturbation and gravitational wave spectra are given by perfect power laws. We compute the corresponding radiation power spectrum by numerically integrating the coupled Einstein, fluid and Boltzmann equations to the present as discussed in Hu et al. (1995b). We carry out a fit to the four-year *COBE* data using the method of White & Bunn (1995) with the customized galactic cut of Kogut et al. (1996). A full description of this calculation will be given by Bunn & White (in preparation).

Defining the spectrum in the manner of equation (1) is extremely useful, because when expressed in terms of $\delta_{\rm H}$ the normalization is *independent* of the present Hubble parameter h to an excellent approximation. Further, the scale of the *COBE* observation is large enough that details of the material content of the Universe are unimportant, making the normalization independent of the baryon density and whether or not the dark matter is multi-component. So the present-day normalization of the power spectrum depends only on the density parameter Ω_0, the tilt of the spectrum n and, if included, the amplitude of any gravitational waves that might influence the *COBE* result.

In the scale-invariant case $\delta_{\rm H}(k)$ is constant, and can be specified on any scale. More generally, it is necessary to specify the scale on which it is quoted, via

$$\delta_{\rm H}^2(k) = \delta_{\rm H}^2(k_0)\left(\frac{k}{k_0}\right)^{n-1} . \qquad (9)$$

We will choose $k_0 = a_0 H_0$, the present Hubble scale, and write $\delta_{\rm H} \equiv \delta_{\rm H}(k_0)$.

We find that we are able to represent the normalization of tilted ΛCDM models by a single fitting function

$$\delta_{\rm H}(n,\Omega_0) = 1.94\times 10^{-5}\ \Omega_0^{-0.785-0.05\ln\Omega_0}\exp\left[f(n)\right] , \qquad (10)$$

where

$$\begin{aligned} f(n) &= -0.95(n-1) - 0.169(n-1)^2 \quad \text{No tensors}\,; &(11)\\ &= 1.00(n-1) + 1.97(n-1)^2 \qquad \text{PLI}\,. &(12)\end{aligned}$$

These fits are accurate to 3 per cent in the range $0.2 < \Omega_0 \leq 1$ and $0.70 < n < 1.2$ (the latter formula though only applying for $n \leq 1$). The statistical uncertainty in all of these numbers is 7 per cent, and there is an additional 3 per cent 'systematic' uncertainty from the choice of galaxy cut; we combine these to obtain a 15 per cent uncertainty at 2σ. The fits are accurate for any reasonable h and $\Omega_{\rm B}$, and for any choice of dark matter content. This extremely simple form is much more easily applied than the tabular data of White & Bunn (1995) and Stompor et al. (1995), as well as applying to a much wider parameter space of models.

Although we normalize all models to the central *COBE* value, we allow for its uncertainty by adding it in quadrature to the relative error of other observations constraining the amplitude.

3.2 Intermediate-scale anisotropies

Potentially the most powerful test structure formation models will have to surpass in the future is the shape of the microwave anisotropy power spectrum on smaller angular scales than those sampled by *COBE*. It is expected that this can be measured to great accuracy by proposed satellite observatories. At present the observational situation is rather uncertain, and so it is best to focus on a single quantity — the height of the peak at degree scales relative to the large angular scale plateau.

In a ΛCDM model, the first peak in the spectrum is located at around $\ell \simeq 220$, with a small dependence on Ω_0

and h. Here ℓ is the multipole number in an expansion of the CMB temperature field: $\Delta T = \sum_{\ell m} a_{\ell m} Y_{\ell m}(\theta, \phi)$ with $Y_{\ell m}(\theta, \phi)$ the usual spherical harmonics. The *COBE* data sample multipoles $\ell = 2$–30, with $\ell \simeq 10$ being the pivot point with changing n. We define a quantity

$$D(\ell) = \frac{\ell(\ell+1)C_\ell}{110 C_{10}}\,, \qquad (13)$$

where the C_ℓ are (as usual) the expected values of the coefficients of the multipole expansion: $C_\ell = \left\langle |a_{\ell m}|^2 \right\rangle$. Focusing on $D(220)$, the predicted value is given by the following fitting function (White 1996):

$$D(220) \simeq 5.1 \left(\frac{220}{10}\right)^{\nu}\,, \qquad (14)$$

where we have formed the degenerate combination of parameters

$$\begin{aligned} \nu \quad \equiv \quad & (n-1) - 0.32\ln(1+0.76r) + 6.8(\Omega_{\rm B}h^2 - 0.016) \\ & -0.37\ln(2h) - 0.16\ln(\Omega_0) - 0.65\tau\,. \end{aligned} \qquad (15)$$

Here r is the tensor to scalar ratio, normalized to $r = 7(1-n)$ for power-law inflation in the $\Omega_0 = 1$ and $n \to 1$ limit as in Davis et al. (1992). This form accurately describes the peak height for $\Omega_0 > 0.3$ and $0.01 \le \Omega_{\rm B}h^2 \le 0.02$. For models with lower Ω_0 the shape of the CMB spectrum is sufficiently different from when $\Omega_0 = 1$ that a case by case comparison is warranted.

The term depending on τ shows the reduction in the peak height due to reionization between $z = 0$ and the last scattering surface. The optical depth to Thomson scattering for ionized fraction $x_{\rm e}$ from $z = 0$ to the reionization redshift $z_{\rm R}$ is

$$\tau = 0.035\,\frac{\Omega_{\rm B}}{\Omega_0}\,h\,x_{\rm e}\left[\sqrt{\Omega_0(1+z_{\rm R})^3 + 1 - \Omega_0} - 1\right]. \qquad (16)$$

The uncertainties in the epoch of reionization and value of $\Omega_{\rm B}h^2$ are the biggest barrier to using the measured height of the peak to constrain the spectral slope.

Present observations, focusing on the *COBE* data and intermediate-scale experiments which probe only around the peak[⋆], provide the constraint $\nu = -0.02 \pm 0.12$ ($\pm 2\sigma$ range). While this result provides a strong constraint in the case of the $\Omega_0 = 1$ models, limiting the degree to which tilt can be used to reduce small-scale power, it is not very constraining for ΛCDM models. The reason for this is that ΛCDM models have a larger amount of power on degree scales than the critical models, giving them more room for a tilt. The upper limit on the height is compromised by the possibility of reionization in these models, which reduces the degree-scale power and is exponentially sensitive to the assumed redshift of reionization. We will only use the lower limit, and make the conservative assumption of no reionization.

We have fixed the baryon density in equation (15) to the central nucleosynthesis value. We include the uncertainty in $\Omega_{\rm B}h^2$ by adding it in quadrature to the observational limits on ν, though this only increases the error bar on ν to ± 0.13. This uncertainty would become greater, however, if one were to allow $\Omega_{\rm B}h^2$ to drift outside the nucleosynthesis range. We note in passing that the next generation of smaller angular scale (mostly interferometer) CMB experiments will allow tighter constraints, and more importantly will suffer less from uncertainties in cosmological modelling.

⋆ We include the new Saskatoon-95 data (Netterfield et al. 1996) in this analysis, marginalizing over the 14 per cent calibration uncertainty. We also include the *COBE*, Python III 3-point, SP94, MAX, and MSAM three-point data which probe predominantly around the first peak; see White (1996) for a discussion of these experiments. We do not perform any foreground subtraction in this analysis.

4 LARGE-SCALE STRUCTURE OBSERVATIONS

Our use of observational data has been described extensively for the case of a critical-density universe in Liddle et al. (1996b), and we will not reproduce the analyses here except where important differences arise when the cosmological constant is introduced. Instead we provide a short summary.

4.1 Galaxy correlations

We use results from the compilation of data carried out by Peacock & Dodds (1994). They supply values of the power spectrum at a selection of different k, which can be used directly to constrain the shape parameter Γ. In their paper they quoted a central value of $\Gamma = 0.255 + 0.32(1/n - 1)$; however, they have noted since (Peacock 1996; Peacock & Dodds 1996) that both non-linear bias and non-linear evolution of the power spectrum may have some effect on short scales. We choose to drop the four shortest scale points and refit. We find $\Gamma = 0.230 + 0.28(1/n - 1)$, where the uncertainty is plus 18 per cent and minus 15 per cent at 95 per cent confidence. The galaxy correlation function data are not useful for constraining the amplitude of the power spectrum.

4.2 POTENT

The bulk velocity flow has long been suggested as one of the main worries for the low-density model, which predicts lower velocities than its critical-density counterpart for the same size of density contrast. However, the *COBE* normalization for low densities is actually much higher, as we have seen, and it will turn out that the velocities give no useful constraint. In fact, at present the bulk velocity constraint is the weakest constraint of all. Additional ways of using the velocity data, which permit stronger constraints not directly interpretable in linear theory, will be discussed in Section 5.3.

We use the measurement of the bulk flow around our position by the POTENT technique (Bertschinger & Dekel 1989), using the Mark III data set (Dekel 1994). This contains the evaluation of the bulk flow in spheres about our position for a variety of radii. However, the bulk flow is sensitive to long wavelengths to a much greater extent than the dispersion of the density field, and consequently in any given realization these data are highly correlated. Because of this, we concentrate on the measurement on a single scale, $40\,h^{-1}$ Mpc. This is obtained observationally first by carrying out a smoothing with a $12\,h^{-1}$ Mpc Gaussian to generate a continuous velocity field, and then velocity reconstruction

is carried out to give the flow corresponding to a top-hat smoothing. The theoretical prediction for the rms bulk flow is therefore

$$\sigma_v^2(R) = H_0^2 f^2(\Omega_0) \int_0^\infty W^2(kR)\, \mathrm{e}^{-(12\,h^{-1}k)^2}\, \frac{\mathcal{P}_0}{k^2} \frac{\mathrm{d}k}{k}, \qquad (17)$$

where $W(kR)$ is the top-hat window given by equation (3) and the factor $f^2(\Omega_0)$ gives the velocity suppression. Often $f(\Omega_0)$ is approximated as $\Omega_0^{0.6}$, which turns out to work well for both open and flat models (Lahav et al. 1991), but for the sake of accuracy we numerically obtain the precise result, following Carroll et al. (1992).

The Mark III POTENT analysis gives the bulk flow in a $40\,h^{-1}$ Mpc sphere as (Dekel 1994)

$$v_{\rm obs}(40\,h^{-1}{\rm Mpc}) = 373 \pm 50 \ {\rm km\,s^{-1}}\,. \qquad (18)$$

The quoted error represents different strategies for coping with sampling gradient bias and is to be considered as an estimate of the systematic error. In addition there is a 1σ random error of 15 per cent. However, both of these are dominated by cosmic variance, which arises because we have only a single measurement which is to be drawn from an ensemble following a χ_3^2 distribution. Modelling this along with the observational errors as in Liddle et al. (1996b), we find that the 95 per cent confidence limits on the estimate of the dispersion are +295 per cent and −47 per cent. The huge asymmetry is caused by the asymmetry of the χ_3^2 distribution, and only the lower limit is of interest to us.

4.3 Cluster abundance

In a recent paper, Viana & Liddle (1996) re-analysed the constraint on the power spectrum from the abundance of large galaxy clusters, using X-ray observations and a calculational technique based on Press–Schechter (1974) theory. Traditionally, the cluster abundance constraint is quoted on the scale $8\,h^{-1}$ Mpc, which in linear theory corresponds more or less to the appropriate mass of a cluster. The result found was

$$\sigma_8 = 0.60\ \Omega_0^{-C(\Omega_0)}\,, \qquad (19)$$

where the fitting function

$$C(\Omega_0) = 0.59 - 0.16\,\Omega_0 + 0.06\,\Omega_0^2 \qquad (20)$$

parametrizes the changing power-law index of the Ω_0 dependence. At $\Omega_0 = 1$, the uncertainty was found to be +32 per cent and −24 per cent. As Ω_0 is decreased, the uncertainty becomes slightly larger; this increase can be expressed by multiplying the uncertainties for $\Omega_0 = 1$ by a factor $\Omega_0^{0.26\log_{10}\Omega_0}$ (though the difference does not really become important until $\Omega_0 < 0.3$).

4.4 Early object formation

For critical-density models of structure formation, such as models with both cold and hot dark matter, an important constraint is whether or not structure can form sufficiently early. The strongest constraints available at the moment come from the amount of gas in damped Lyman alpha systems, which for those models imposes new constraints on parameter regions not excluded by other data.

For low-density models, whether open or spatially flat, these constraints are much less of a concern, for the reason that structure has been growing much more slowly recently, and hence one automatically finds that at high redshift structure should be more advanced than in the critical-density case. Nevertheless, for completeness we perform here a calculation of the constraint arising from damped Lyman alpha systems, following the strategy based on Press–Schechter theory utilized by Liddle et al. (1996a) to extend the usual calculation to the open universe case.

The contribution of the baryons present in damped Lyman alpha systems to the mean cosmological mass density at some redshift z is given by (e.g. Padmanabhan 1993, p. 347)

$$\rho_{\rm DLAS}(z) = \mu m_{\rm p}\, \langle \bar{N} \rangle\, \frac{\mathrm{d}N}{c\mathrm{d}t}\,, \qquad (21)$$

where μ is the baryonic mean molecular weight, $m_{\rm p}$ is the proton mass, $\langle \bar{N} \rangle$ is the mean H I column density as a function of redshift and $\mathrm{d}N/c\mathrm{d}t$ is the number density of damped Lyman alpha systems per unit length interval at that redshift. As $\mathrm{d}N/\mathrm{d}z$ is a measurable quantity, the only dependence of $\rho_{\rm DLAS}(z)$ on the assumed cosmology comes from $\mathrm{d}z/c\mathrm{d}t$. Using the Friedmann equation it is straightforward to show that, for the cosmological models we are interested in, the amount of baryonic matter deduced from observation at a given z is given by the following function of Ω_0 :

$$\Omega_{\rm DLAS}(\Omega_0, z) = \Omega_{\rm DLAS}(\Omega_0 = 1, z) \sqrt{\frac{\Omega_0}{\Omega(z)}}\,. \qquad (22)$$

For our cosmology, $\Omega(z)$ is given by equation (7), though the above result applies even to non-flat cosmologies with only matter plus a cosmological constant provided that the appropriate $\Omega(z)$ is used. Expressing $\Omega_{\rm DLAS}(\Omega_0, z)$ in terms of its value assuming a critical-density cosmology is useful since most quoted observational results for $\Omega_{\rm DLAS}(z)$ assume such a cosmology.

Estimates of the abundance of baryons in damped Lyman alpha systems have recently been provided at redshifts 3 and 4 by Storrie-Lombardi et al. (1995), the redshift 3 data giving a somewhat lower result than the much used earlier data of Lanzetta, Wolfe & Turnshek (1995). The constraints from the redshift 3 and 4 points are very similar, and we will use the latter. Using the above expression we then have

$$\Omega_{\rm DLAS}(z=4) = (0.0011 \pm 0.0002)\, h^{-1} \sqrt{\frac{\Omega_0}{\Omega(z=4)}}\,, \qquad (23)$$

where the uncertainty is 1σ. As mentioned before, $\Omega(z)$ is given by equation (7) and at redshift 4 is very close to unity for any reasonable Ω_0. Expression (23) conservatively assumes all the gas to be in the neutral state.

Since we assume all the dark matter to be cold, a reasonable hypothesis is that the total density of the systems is larger than the baryonic density by a factor $\Omega_0/\Omega_{\rm B}$. Then the fraction of mass which is in bound objects of mass at least the typical mass M is given by

$$f(>M, z=4) > (0.069 \pm 0.021)\, h \sqrt{\frac{\Omega_0}{\Omega(z=4)}}\,, \qquad (24)$$

where we have interpreted the error on $\Omega_{\rm B}h^2$ in Copi et al. (1995b) as 95 per cent confidence and added it in quadra-

ture. Using Press–Schechter theory (Press & Schechter 1974), the theoretical prediction for this quantity is

$$f(>M,z) \equiv \frac{\Omega(>M(R),z)}{\Omega(z)} = \mathrm{erfc}\left(\frac{\delta_{\rm c}}{\sqrt{2}\,\sigma(R,z)}\right), \qquad (25)$$

where $\delta_{\rm c}$ is a threshold parameter to be fixed via N-body simulations and the smoothing to obtain $\sigma(R)$ is carried out via the top-hat window of equation (3).

The most conservative assumption is that the damped Lyman alpha systems have not fully collapsed along all their axes. This corresponds to a lower choice of threshold than the usual 1.7, so, following Liddle et al. (1996b), we adopt $\delta_{\rm c} = 1.5$ (Monaco 1995). The minimum mass of the damped Lyman alpha systems (from the assumption that they eventually give rise to rotationally supported discs as indicated by lower redshift observations (Lanzetta et al. 1995)) is taken as $10^{10} h^{-1} {\rm M}_{\odot}$ (Haehnelt 1995).

5 MISCELLANEOUS CONSTRAINTS

In this section we consider other constraints which motivate the preferred choice of Ω_0. They either fall outside the description of large-scale structure, or correspond to constraints from large-scale structure that we have insufficient technology to compute ourselves and must take at face value from other papers.

5.1 Age of the Universe

The age of a flat universe is given by†

$$H_0 t = \frac{2}{3}\frac{1}{\sqrt{1-\Omega_0}} \ln\left[\frac{1+\sqrt{1-\Omega_0}}{\sqrt{\Omega_0}}\right]. \qquad (26)$$

At fixed H_0, lowering Ω_0 increases the age. However, as already noted for the case of an open universe (Liddle et al. 1996a), observations such as the galaxy correlation function more or less fix the combination $\Omega_0 H_0$, rather than H_0 itself. With that combination fixed, lowering Ω_0 actually leads to a reduction of the age of the model universe (though a less severe one than in the open case). Consequently, ΛCDM models that fit other observations are *younger* the lower the density.

Observational age determinations remain controversial, perhaps solely because the older estimates are in conflict with almost any cosmology (and most seriously with *low*-density cosmologies, as seen in our figures). However, a recent analysis by Chaboyer et al. (1996) (see also Bolte & Hogan 1995; Jiminez et al. 1996) suggests a 95 per cent lower limit of 12 Gyr, which is weak enough to leave most possibilities intact.

5.2 Baryons in clusters

The piece of evidence which is presently providing the strongest push for researchers to consider low-density models is the question of the baryon density in clusters (White & Frenk 1991; White et al. 1993). Typical quoted values suggest that about 20 per cent of material in clusters is in the form of baryons, primarily as hot intracluster gas. Taken at face value, this is consistent with standard nucleosynthesis only if the total density is substantially less than critical.

Although a pressing concern, there remain some potential problems with the interpretation of the observations (see for example the summary by Steigman & Felten 1995). A standard argument is that clusters are such large objects that the fractional baryon density within them must be representative of the Universe as a whole; however, individual measurements of the fractional baryon density are not all consistent with each other on a cluster-by-cluster basis (e.g. White & Fabian 1995), so unless the observational errors have been somewhat underestimated this assumption must fail at some level. Nevertheless, this doesn't seem to be too important an uncertainty. For example, an unreasonably crude method of bringing the White & Fabian (1995) results into agreement with one another is to throw out the clusters with the highest four or five determinations; even this only reduces the best-fitting baryon density by about 10 per cent. Retaining all of their preferred sample of 13 clusters, White & Fabian find

$$\frac{\Omega_{\rm B}}{\Omega_0} = 0.05^{+0.03}_{-0.015}\, h^{-3/2}, \qquad (27)$$

at the 95 per cent confidence level. A further question concerns whether or not the mass determinations (of either the gas or the total mass) are sufficiently sophisticated: for example, in a recent paper Gunn & Thomas (1996) carried out an analysis modelling the cluster gas as a multiphase medium, and found that the inferred fractional baryon density can be reduced by a quarter or even a half. Finally, the status of standard nucleosynthesis theory has been less static recently than for many years, with the upper limit on the baryon density rising substantially. Copi et al. (1995a,b) advocate the central value $\Omega_{\rm B} h^2 = 0.016$ that we are adopting, and their 95 per cent confidence upper limit is 0.024 (see also Kernan & Krauss 1994; Sasselov & Goldwirth 1995; Krauss & Kernan 1995; Hata et al. 1995, and for an opposite opinion see Hogan 1995). Some recent deuterium abundance measurements in quasar absorption systems suggest even higher baryon fractions (Tytler, Fan & Burles 1996; Burles & Tytler 1996).

Ignoring mass determination uncertainties beyond those accounted for by White & Fabian (1995), in combination the cluster and nucleosynthesis limits imply a central value $\Omega_0 = 0.32 h^{-1/2}$. Taken at face value the error bars exclude the critical-density case for any reasonable h (the 95 per cent upper limit being $0.53 h^{-1/2}$), but in our view, shared by Babul & Katz (1993) and Gunn & Thomas (1996) (see also Balland & Blanchard 1995), the theoretical uncertainties remain great enough that it would be extremely premature to abandon critical-density models completely. It is clear however that present interpretation of this vital constraint does favour values of Ω_0 in the range between 0.2 and 0.6.

5.3 Velocity flows

A direct analysis of the velocity flow data has been carried out by Nusser & Dekel (1993) and Bernardeau et al. (1995).

† There is an equivalent expression where the logarithm is replaced by $\sinh^{-1}\left(\sqrt{(1-\Omega_0)/\Omega_0}\right)$.

By analysing statistics of the velocity field in the weakly non-linear regime, they conclude that Ω_0 must exceed 0.3 at the 95 per cent confidence level. The same result has also been obtained from an analysis of velocity outflows in void regions (Dekel & Rees 1994). Summaries of these results are given in reviews by Dekel (1994) and Strauss & Willick (1995). The parameter region excluded by this constraint has only minimal overlap with our allowed regions, and always in a region where the age of the universe is anyway suspiciously low. So our analysis independently supports the conclusion of these papers.

The results of those methods are quoted independently of any assumptions about the underlying power spectrum (though the former method does have some dependence on the assumption made). It would be expected that if the analyses were repeated with specific power spectra input, then the constraints would strengthen[‡].

5.4 Lensing limits on Λ

An extremely comprehensive account of lensing limits on the cosmological constant has recently been presented by Kochanek (1995). He concludes that the 95 per cent confidence limit for flat universes is $\Omega_0 > 0.34$, a constraint which is similar to that obtained from velocities. However, he notes that this limit is sensitive to several possible systematic biases, and it can be relaxed to some extent by the introduction of extinction due to dust obscuration in the lensing population of mainly E/S0 galaxies.

5.5 Type Ia supernovae

Prospects now look extremely good for applying a classic cosmological test, using type Ia supernovae as standard candles at very large distances in order to study the deceleration of the Universe. Preliminary results from the Supernova Cosmology Project (Perlmutter et al. 1996) favour a decelerating Universe rather than an accelerating one, which if confirmed will give a stronger constraint than the lensing limits of the previous subsection. A decelerating flat universe requires $\Omega_0 > 2/3$. With many more supernovae already being analysed, this is perhaps the most promising route of all towards constraining low-density flat universes.

5.6 Galaxy counts

Faint galaxy counts in principle offer a fairly direct probe of the geometry of the Universe to high redshifts, but in practice the separation of geometry from evolution of the galaxy population has proven difficult. A recent analysis using *HST* observations (Driver et al. 1996) concludes with some definiteness that for a flat universe Ω_0 must exceed 0.2, which is weaker than other constraints. A similar constraint had already been obtained from deep number counts by Gardner, Cowie & Wainscoat (1993). However, Driver et al. (1996) add that with further assumptions concerning correctness of current morphological classifications, that limit can already be dramatically strengthened to $\Omega_0 > 0.8$ for a flat geometry. Further development of this constraint is clearly a promising avenue.

6 RESULTS

Our results are shown in Figs 1 and 2, which indicate the microwave background and large-scale structure constraints, along with some age contours for reference. Let us first discuss Fig. 1, which shows the situation without gravitational waves.

Fig. 1 shows that incorporating the possibility of tilt greatly increases the space of viable parameters. By far the most important constraint is the shape of the galaxy correlation function; this constraint is reinforced by the cluster abundance constraint which lies in a similar region but is typically weaker. For low n, the allowed region is then further trimmed by the damped Lyman alpha system abundance. The POTENT constraint is weaker than the others in all parts of parameter space and we omit it for clarity. At low Ω_0, models start to become dangerously young.

One can go to very high n in these models. Although we haven't indicated it in a plot, we have found that there remains an allowed region even at $n = 1.4$. However, once $n > 1.3$, a strong anti-bias is necessary unless h is well below 0.5. Also, such models predict a huge Doppler peak, and require a redshift of reionization which is just right to reduce the large peak down to the standard size.

In almost all regions of parameter space the height of the Doppler peak is not constraining. The exception is when $n = 0.8$, where it begins, quite dramatically, to cut off the high-h region. To avoid confusion through plotting yet another line, we have indicated this simply by not shading the region (centred around $\Omega_0 = 0.5$, $h = 0.75$) that is allowed by other constraints but not the Doppler peak height. The exclusion is because for these parameters the peak is not high enough to fit the observations. As n is decreased, the Doppler peak constraint rapidly becomes very strong; already by $n = 0.75$ there is no allowed region at all. Note though that the value of n at which this happens would be changed were the baryon density allowed to be substantially higher than indicated by nucleosynthesis.

The main problem with the Doppler peak constraint is that the upper limit cannot be used due to the possibility that reionization might bring the peak back down to an acceptable height. However, some regions of otherwise permitted parameters do require specific assumptions about reionization and we will discuss this shortly.

Except in rather extreme circumstances, the region allowed by these large-scale structure constraints is not further reduced by including the miscellaneous constraints from bulk velocity flows, gravitational lensing and galaxy counts that we discussed in Section 5, which favour $\Omega_0 > 0.3$. Rather, our results provide independent support to the conclusions of those analyses. Type Ia supernovae (Perlmutter et al. 1996) seem the most promising route to a stronger constraint in the immediate future.

In Fig. 1, we have shaded the allowed regions in two

‡ After our paper was submitted, analyses based on computation of the power spectrum of mass fluctuations directly from the POTENT Mark III catalogue were produced (Kolatt & Dekel 1995; Zaroubi et al. 1996), including comparison with a range of cosmological models. Where they overlap, our results seem in good agreement with theirs.

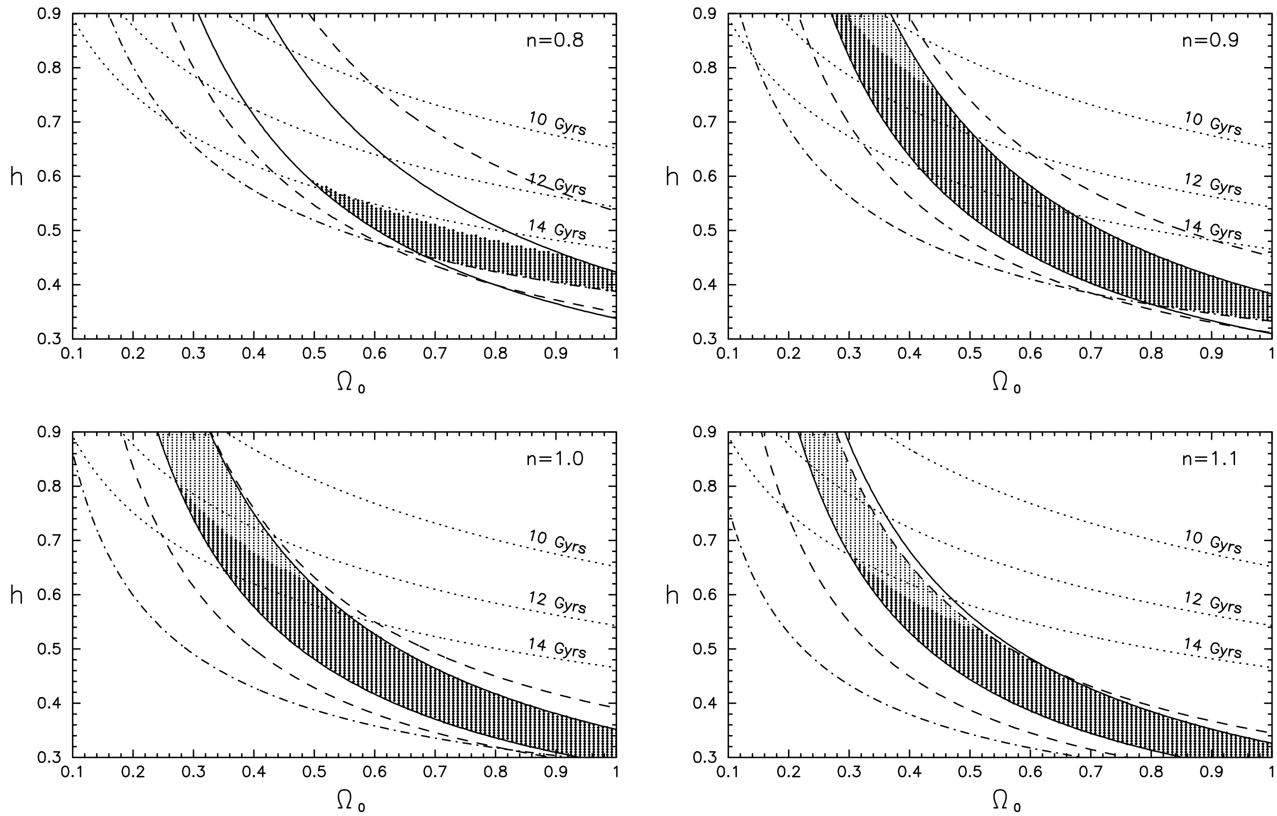

Figure 1. The constraints plotted in the Ω_0–h plane, for different n in the case without gravitational waves. All constraints are plotted at 95 per cent confidence. The models are normalized to the *COBE* data. The solid lines are limits from the shape parameter, the dashed lines from the cluster abundance and the dot-dashed lines from the abundance of damped Lyman alpha systems. The POTENT constraint is not shown as it is weaker than all the other constraints. Contours of constant age are shown as dotted lines. The allowed region is shown with two different shadings, both highlighting the parameter space not excluded at more than 95 per cent confidence on any single piece of data. The lighter shading shows models where the optical galaxies have to be anti-biased at $8\,h^{-1}$ Mpc. The unshaded region in the $n = 0.8$ plot which is allowed by all plotted data is excluded by Doppler peak height.

different styles, to indicate whether anti-biasing of optical galaxies at $8\,h^{-1}$ Mpc is required or not. In doing this, we take the *COBE* normalization to its 95 per cent lower limit, so the light shaded regions definitely require anti-biasing. Whether this is excluded on any particular observational grounds is unclear. Motivated by Klypin et al. (1995), we have also tried to calculate to what extent anti-bias is required at scales presently in the non-linear regime. We choose to consider the APM power spectrum measurement (Baugh & Efstathiou 1994) at $1\,h^{-1}$ Mpc. Using analytic approximations to estimate the non-linear power spectrum (Hamilton et al. 1991; Peacock & Dodds 1994; Jain, Mo & White 1995; Peacock & Dodds 1996) for the models we are interested in, we can determine which require an anti-bias at this scale. As it turns out, the resulting constraint is not much stronger than assuming no anti-bias at $8\,h^{-1}$ Mpc using the linear power spectrum. In any case it seems less clear that anti-biasing is physically forbidden so far into the non-linear regime. The question of whether anti-bias is permitted will perhaps be accessible to new hydrodynamical N-body simulations, which might indicate whether or not forbidding anti-bias at $8\,h^{-1}$ Mpc can really be adopted as a firm constraint[§].

Our conclusion in the case of no gravitational waves is that the large-scale structure data alone permit Ω_0 as low as about 0.30, and show no preference for any particular value above that. However, at critical density the required Hubble parameter is very low (though see the more extensive discussion in Liddle et al. (1996b) which also considers adding a hot dark matter component). The ΛCDM model is compatible with observations for a wide range of values of the tilt.

Fig. 2 shows what happens if gravitational waves are introduced in accordance with the power-law inflation model. This model requires $n < 1$ so we only show two values. The introduction of gravitational waves leads to a much smaller allowed region than in their absence, with $n = 0.9$ the smallest allowed value. At $n = 0.9$, the shape and cluster constraints leave only a very narrow strip, which is then totally excluded by the Doppler peak constraint. For $n = 0.95$ there

§ Also see Kauffmann, Nusser & Steinmetz (1995) for a recent discussion of the physical origin of biasing; their analysis does not favour anti-biasing.

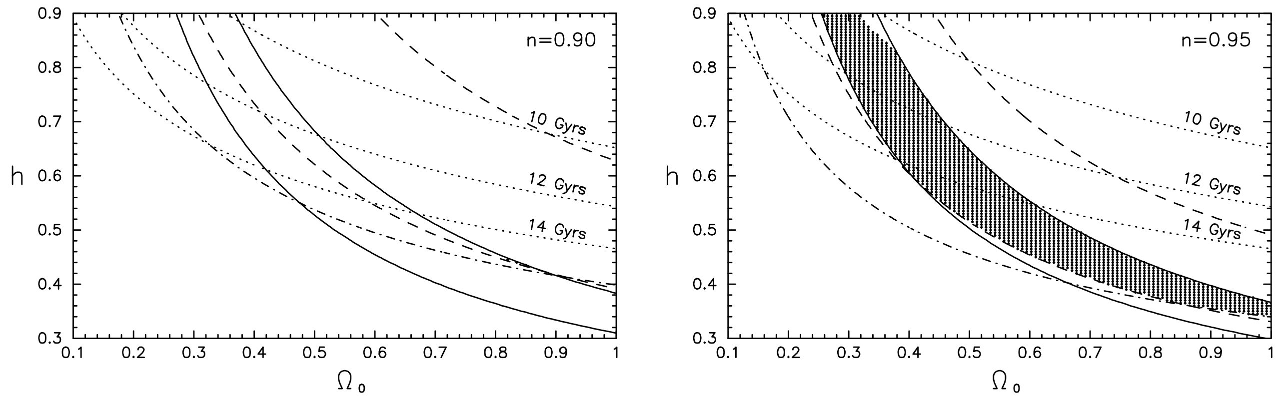

Figure 2. As Fig. 1, but with gravitational waves as given by power-law inflation. In each case where only a single line appears, the allowed region is to the upper right. In both of these plots, the unshaded region which is allowed by all plotted data is excluded by Doppler peak height.

is an allowed region. The limit $n > 0.9$ in the power-law inflation model is stronger than that given by Liddle et al. (1996b) in the critical-density case, because here we consider the additional constraint coming from the Doppler peak.

Although we will not fully investigate the issue here, the Doppler peak constraint does have some implications for reionization. Because ΛCDM models at low density have a very high normalization (especially at high redshift), early reionization is more likely in these models than in other cosmologies (Liddle & Lyth 1995). However, the physical understanding of the reionization process remains poor enough that most estimates of the reionization redshift can be interpreted only as upper limits.

We'll restrict ourselves to the case with no gravitational waves, and for simplicity assume the central nucleosynthesis value for $\Omega_{\rm B}$ (to which the results are quite sensitive). In some regions of parameter space, any early reionization would remove the Doppler peak sufficiently to be incompatible with observation, while in others early reionization is necessary to bring it down to acceptable levels. Using the formulae given earlier, we find the following results. For $n \geq 1.1$, reionization at any $z_{\rm R}$ less than 15 is not enough to allow any interesting models. For lower n, reionization becomes less necessary in order to lower the Doppler peak; indeed, it becomes a worry that it may lower it too much. If $n = 0.8$, then $z_{\rm R} \geq 15$ is sufficient to exclude all the otherwise allowed region. A full examination of the relationship between large-scale structure constraints and the required reionization properties, taking into account the uncertainty in the baryon density, is clearly warranted.

7 CONCLUSIONS

Our results extend previous work both in terms of the range of parameter space studied and in terms of the observations considered. By including the possibility of tilt and gravitational waves, we have explored the parameter space available for inflationary-based ΛCDM models. Recent work (Krauss & Turner 1995; Ostriker & Steinhardt 1995; Bagla et al. 1995) has favoured flat low-density models primarily not for large-scale structure reasons, but rather as a resolution of two separate conflicts, the first being age verses the Hubble constant and the second being the cluster baryon fraction verses nucleosynthesis. We have shown that while compatibility with large-scale structure does not in itself select favoured values of Ω_0, it is compatible with low-density ΛCDM models.

It is interesting to compare these results with the open CDM case explored using the same techniques in Liddle et al. (1996a). The principal difference is that at low Ω_0 the *COBE* normalization is very different, so that in the open case anti-bias is not necessary. However, other observations, such as the cluster abundance, shift as well to maintain a substantial allowed region. The allowed regions are fairly similar in the ΛCDM and open CDM cases, with no particular preference for either on present data, though the open models tend to prefer a slightly higher value of n. In going to a ΛCDM model, there is only a very modest gain in age relative to an open CDM model, so the principal motivation for introducing the cosmological constant is in maintaining consistency with standard inflationary models rather than because of an age crisis.

Returning to our discussion of the ΛCDM model, an extremely useful result is the fitting function, equation (10), which allows a quick normalization of all Λ models, since to an excellent approximation it is independent of the parameters h and $\Omega_{\rm B}$ and the presence of any component of hot dark matter. We have also made the first detailed comparison of these models with the observational situation concerning intermediate-scale CMB anisotropies; though observations there are still at a very primitive level compared with what one hopes will become possible in the future, already one can obtain new constraints against regions of parameter space involving substantial gravitational waves, which are viable against all other present observations. We have also noted that certain areas of parameter space are viable against this constraint only if reionization occurred suitably; in some regions it is required to happen fairly early, while in others it is forbidden from doing so.

In the case of a scale-invariant spectrum with no gravitational waves, our results are in broad agreement with those

obtained by other authors. The main challenge from large-scale structure for the low-density versions of these models continues to be the question of whether it is reasonable to believe that optical galaxies are un- or even anti-biased. We have included more constraints than the earlier treatments but for scale-invariant spectra they are not more constraining.

By extending the parameter space to include tilt and gravitational waves, we have found a large region compatible with present data. Amongst low-density models, it seems Ω_0 in the range 0.4 to 0.5 is better than $\Omega_0 = 0.3$ (continuing a historical upward trend in the preferred 'low-density' Ω_0), as is also the case for open CDM models (Liddle et al. 1996a). However, large-scale structure studies do not look particularly promising for lifting the degeneracy of allowed Ω_0 values; the types of evidence we discussed in Section 5 appear more promising for delivering an ultimate verdict for or against the cosmological constant.

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